MECHANISM OF HEAT EXCHANGE BETWEEN SURFACES AND A FLUIDIZED BED

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Heat exchange between surfaces and fluidized beds is analyzed in terms of a solution for a problem with boundary conditions of the 1-st and 4-th kinds.

In the transfer of heat between a surface and a fluidized bed heat is transmitted from the wall via slugs (packets) of particles periodically forced apart by gas bubbles passing between them [1–4], resulting in variation of the instantaneous heat-exchange coefficient from its maximum value to virtually zero [2]. Along with the thermal resistance of the unsteadily heated packet of particles, it had been suggested in [3] to consider in the heat exchange process the additional "contact" thermal resistance to heat transfer from the surface to the packet at the packet boundary. This resistance is due to the increased porosity of that part of the bed in close proximity to the wall. This makes it necessary to introduce the assumption of contact-resistance constancy with respect to time, to take into consideration the heat exchange between individual particles and the wall, etc. [3]. It is expedient to analyze the heating of a packet which consists of two layers: the first layer corresponds to the region of higher porosity, and the second layer corresponds to the remaining packet volume of constant porosity. The problem reduces to consideration of the unsteady heating of two bodies: bounded and semibounded rods exhibiting diverse thermal properties as a result of the difference in the porosities of these two regions. At the initial instant of time (the instant of contact between the packet and the wall) the temperature of the left side of the bounded rod is instantly increased to the wall temperature t_w which remains constant throughout the heating period (boundary condition of the 1-st kind). The exchange of heat between the rods is subject to the Fourier law (boundary condition of the 4-th kind), which corresponds fully to the physical conditions of the problem. The initial temperature of the two rods is the same, and equal to the temperature t_0 of the bed. Width R of the bounded rod corresponds to the width of the higher-porosity zone.

The problem is formulated as follows [5]:

$$\frac{\partial \Theta_1(x,\tau)}{\partial \tau} = a_1 \frac{\partial^2 \Theta_1(x,\tau)}{\partial x^2}, \quad \tau > 0, \quad 0 < x < R, \tag{1}$$

$$\frac{\partial \Theta_2(x,\tau)}{\partial \tau} = a_2 \frac{\partial^2 \Theta_2(x,\tau)}{\partial x^2}, \quad \tau > 0. \quad R < x < \infty, \tag{2}$$

$$\Theta_1(x, 0) = \Theta_2(x, 0) = 0, \tag{3}$$

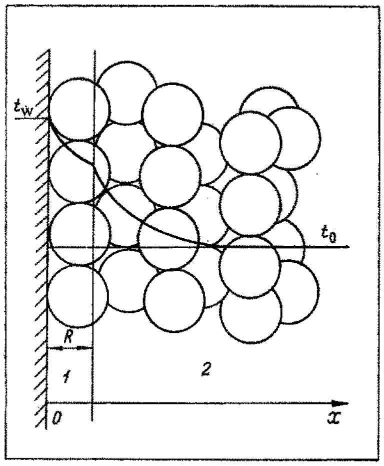
$$\Theta_1(R, \tau) = \Theta_2(R, \tau), \tag{4}$$

$$\lambda_1 \frac{\partial \Theta_1(R,\tau)}{\partial x} = \lambda_2 \frac{\partial \Theta_2(R,\tau)}{\partial x}, \tag{5}$$

$$\Theta_1(0, \tau) = \Theta_w = \text{const}, \tag{6}$$

$$\Theta_2(\infty, \tau) = 0, \tag{7}$$

where $\Theta_1 = t_1 - t_0$; $\Theta_2 = t_2 - t_0$; $\Theta_w = t_w - t_0$.



Heat exchange between a surface and a packet of particles in a fluidized bed. 1) Zone of contact with the wall; 2) bulk of fluidized bed.

A solution of this problem is given in [5]. For the temperature of zone 1 it is written as follows:

$$\Theta_1 = \Theta_w \left\{ \operatorname{erfc} \frac{x}{2\sqrt{a_1 \tau}} - h \sum_{n=1}^{\infty} h^{n-1} \times \left[\operatorname{erfc} \frac{2nR - x}{2\sqrt{a_1 \tau}} - \operatorname{erfc} \frac{2nR + x}{2\sqrt{a_1 \tau}} \right] \right\}. \tag{8}$$

Here

$$h = \frac{1-k}{1+k_\varepsilon}; \quad k_\varepsilon = \sqrt{\frac{\lambda_1 c_1 \gamma_1}{\lambda_2 c_2 \gamma_2}}.$$

The density of the heat flux from the wall to rod 1 (in fact, to the packet) is

$$q = -\lambda_1 \frac{\partial \Theta_1(0\ \tau)}{\partial x}. \tag{9}$$

From (8) and (9) we have

$$q = \frac{\lambda_1 \Theta_w}{\sqrt{\pi a_1 \tau}} \left[1 + 2h \sum_{n=1}^{\infty} h^{n-1} \exp\left(-\frac{n^2 R^2}{a_1 \tau} \right) \right]. \tag{10}$$

Values for Coefficients of Heat Transfer from a Wall to a Fluidized Bed

Material of particles	Particle size, m	Gas velocity, m/sec	α_e, W/m^2 × × deg	α_1, W/m^2 × × deg.	α_2, W/m^2 × × deg.	δ_1, %	δ_2, %
Glass	$0.32 \cdot 10^{-3}$	0.2	367	300	304	17.89	16.9
"	"	0.411	339	272	252	19.8	25.5
"	$0.15 \cdot 10^{-3}$	0.0877	525	420	440	19.79	16.23
"	"	0.122	497	381	397	23.54	20.0
"	"	0.317	434	387	401	10.63	7.79
"	"	0.39	407	332	344	18.18	15.55
"	"	0.454	384	322	329	16.15	13.8
"	"	0.543	366	321	327	12.37	10.17
"	$0.104 \cdot 10^{-3}$	0.076	568	450	480	20.64	15.09
"	"	0.107	538	444	472	17.42	12.12
"	"	0.138	520	420	445	19.39	14.53
"	"	0.201	491	403	425	17.95	13.45
"	"	0.299	478	385	405	19.26	15.52
"	"	0.363	457	379	395	17.43	13.69
"	"	0.451	464	388	405	16.21	12.75
"	"	0.533	464	396	410	14.25	11.28
"	$0.07 \cdot 10^{-3}$	0.07	680	511	550	24.71	19.31
"	"	0.101	642	493	526	23.4	18.12
"	"	0.133	621	485	513	22.15	17.15
"	"	0.198	616	464	493	24.82	19.95
"	"	0.305	604	486	520	19.31	14.03
"	"	0.384	672	484	512	28.02	23.68
"	"	0.457	650	509	539	21.79	17.28
"	"	0.539	702	525	555	25.22	20.95
"	$0.043 \cdot 10^{-3}$	0.155	870	579	622	33.89	28.36
"	"	0.192	852	730	782	14.86	8.26
"	"	0.287	850	660	715	21.98	15.68
"	"	0.378	860	625	678	27.15	21.34
"	"	0.482	808	641	690	20.92	14.73
Polystyrene	$0.14 \cdot 10^{-3}$	0.0686	292	262	264	10.4	9.91
"	"	0.0945	276	259	260	13.96	13.56
"	"	0.126	292	229	231	21.63	21.26
"	"	0.197	272	234	235	13.98	13.46
"	"	0.283	272	231	232	15.13	14.74
"	"	0.344	288	245	246	15.13	14.82
"	"	0.439	299	263	264	11.97	11.59
"	"	0.500	304	253	254	17.04	16.71
"	$0.104 \cdot 10^{-3}$	0.0686	321	295	297	7.78	7.12
"	"	0.0945	325	280	282	14.08	13.5
"	"	0.126	330	256	259	21.94	21.5
"	"	0.197	335	266	268	20.6	20.16
"	"	0.277	351	262	266	24.8	24.34
"	"	0.344	344	281	282	18.43	17.96
"	"	0.436	339	275	276	18.9	18.44
"	"	0.515	351	304	306	13.24	12.75

On the assumption that heat exchange with the bubble can be neglected, the mean value of the coefficient of heat exchange between the wall and the packet is [2, 3]

$$\alpha = \frac{1-f_0}{\tau_c \Theta_w} \int_0^{\tau_c} q \, d\tau. \tag{11}$$

Substituting the value of q from (10) into (11) and integrating, after transformation we obtain

$$\alpha = \frac{2(1-f_0)\lambda_1}{\tau_c} \left\{ \sqrt{\frac{\tau_c}{\pi a_1}} + 2h \sum_{n=1}^{\infty} h^{n-1} \left[\sqrt{\frac{\tau_c}{\pi a_1}} \times \right.\right.$$
$$\left.\left. \times \exp\left(-\frac{n^2 R^2}{a_1 \tau_c}\right) - \frac{nR}{a_1} \operatorname{erfc}\left(\frac{nR}{\sqrt{a_1 \tau_c}}\right) \right] \right\}. \tag{12}$$

To calculate α by means of formula (12) it is necessary to know the thermal characteristics of the packet in zones 1 and 2. Owing to the lack of more general formulas, to estimate the porosity of the zone in contact with the wall we use the empirical relationship given in [6] for large particles (1.0–4.0 mm). It was shown in that paper that the porosity in the stationary layer of particles in contact with the wall is

$$\varepsilon = 1 - 1.25(1-\varepsilon_2)\left(\frac{x}{d}\right)^{0.378}. \tag{13}$$

The width of the higher-porosity zone is equal to 0.56d. The corresponding porosity of an inner column of the bed is [6]

$$\varepsilon = 1 - 1.15(1-\varepsilon_2)\left(\frac{x}{d}\right)^{0.378}. \tag{14}$$

Here the width of the higher-porosity zone is equal to 0.7d. In expressions (13) and (14) x denotes the instantaneous distance of a point from the wall. In the remaining part of the bed (packet) the porosity is constant and equal to ε_2. Integrating expression (13) from 0 to 0.56d, and expression (14) from 0 to 0.7d, we obtain in both cases the same equation for the mean porosity in zone 1,

$$\varepsilon_1 = 1 - 0.73(1-\varepsilon_2). \tag{15}$$

This value can be used to determine the thermal characteristics of zone 1. Values of the specific heat for zones 1 and 2 can be assumed equal, since the specific heat of the gas, compared to that of the particles, is negligibly small, and variation of the porosity has virtually no effect on this characteristic of the packet. The thermal conductivity coefficients λ_1 and λ_2 can be determined from the formula proposed in [4] to approximate the heat-conduction curve in a stationary particulate bed

$$\frac{\lambda_i}{\lambda_g} = 1 + \frac{(1-\varepsilon_i)(1-\lambda_g/\lambda_m)}{\lambda_g/\lambda_m + 0.28\varepsilon_i^{0.63(\lambda_m/\lambda_g)^{0.18}}} \quad (i = 1, 2). \tag{16}$$

The thermal diffusivity in each zone will then be:

$$a_1 = \frac{\lambda_1}{0.73 c_m \gamma_m (1-\varepsilon_2)}, \tag{17}$$

$$a_2 = \frac{\lambda_2}{c_m \gamma_m (1-\varepsilon_2)}, \tag{18}$$

and

$$k_\varepsilon = \sqrt{\frac{0.73\lambda_1}{\lambda_2}}. \tag{19}$$

As an example, calculations were carried out with f_0 and τ_c taken from [2], and compared with the heat-exchange coefficient obtained in the same paper (table). This was done with data from all individual experiments on which the set of magnitudes α, f_0, $(1 - f_0)/\tau_c$ cited in [2] were based.

Heat-exchange coefficients α_2 were also calculated with the use of values λ_2 obtained experimentally [2]. Thus, α_1 and $\delta_1 = (\alpha_e - \alpha_1)/\alpha_e$ were derived for λ_2 calculated by formula (16); and values of α_2, and $\delta_2 = (\alpha_e - \alpha_2)/\alpha_e$ for values of the thermal conductivity coefficient λ_2 taken from experiments [2]. The calculations were carried out with the use of 10 terms of the summation in the right side of Eq. (12). Adequate accuracy can, however, be obtained when the number of summation terms is limited to 2–4.

The analysis of results shows a satisfactory correlation between the calculated and experimental data, although calculated values are somewhat lower. The apparent discrepancy may be due to several factors: the empirical equation for porosity in the zone in contact with the wall was obtained for larger particles; the approximate equation (16) results in a certain deviation from the curve of effective heat conduction in a stationary bed [4]; the convective heat transfer between particles was also neglected.

It should be noted that the above analysis relates to comparatively highly disperse systems, and was carried out on the assumption that the surface area of heat exchange between particles and the gas filtering through the packet is sufficiently large to ensure a close link between the temperature fields of particles and gas. As shown in [7], Eqs. (1) and (2), which are valid for continuous media, may also be used for disperse systems. An evaluation of the error introduced by the above assumptions would, however, require detailed calculation supported by all necessary experimental data (including the coefficient of heat transfer between particles and gas in each zone).

A more precise definition of all factors considered here will make it possible to obtain values of coefficients which would be closer to the actual.

Thus, knowledge of the hydrodynamics (f_0, τ_c), of the bed structure $(\varepsilon_1, \varepsilon_2)$, and of the extent of the relationship (α_g) between the temperature fields of the particles and the gas would completely define the intensity of heat transfer between a surface and a bed of a fluidized particulate material.

NOTATION

t_0 and t_w are the bed and the wall temperatures, respectively; x is the instantaneous distance from the wall; R is the thickness of the rod in contact with the wall; a_1, λ_1, c_1, γ_1, and ε_1 are, respectively, the thermal diffusivity, the thermal conductivity, the specific heat, the density, and the porosity of the zone in contact with the wall; a_2, λ_2, c_2, γ_2, and ε_2 are, respectively, the thermal diffusivity, the thermal conductivity, the specific heat, the density, and the porosity of the bulk of the bed; τ is the time, q is the heat flux, λ_m and λ_g are the thermal conductivities of the material of particles, and of the gas, respectively; α is the mean value of the coefficient of heat transfer from the wall to the stationary bed; f_0 is that part of the time during which the surface is in contact with the gas bubbles; τ_c is the time of contact between the bed and the surface; ε is the instantaneous porosity of the zone in contact with the wall; d is the particle diameter; c_m is the specific heat of the material of the particles; γ_m is the density of the material of the particles; α_e is the experimental value of the heat-transfer coefficient; and α_g is the coefficient of heat transfer between the particles and the wall.

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